Extra credit 1

(Dated: 2 April, 2017)

The rocket equation determines the maximum change in velocity of a rocket that propels itself by ejecting mass at a fixed velocity v_e in the absence of external forces. You can imagine the common picture of a rocket that accelerates by burning solid or liquid fuel and directing the combusting fuel out one end of the rocket to propel it in the other direction. The change in velocity is given by

$$v_f - v_i = -v_e \ln\left(\frac{m_f}{m_i}\right) \tag{1}$$

where v_f is the final velocity of the rocket, v_i is the initial velocity, m_f is the final mass of the rocket (plus any remaining fuel) and m_i is the initial mass of the rocket and fuel.

1. [1 percentage point] Derive the maximum change in velocity in case the exhaust velocity is proportional to remaining fuel (instead of a constant). You can think of this is as a model of water rocket, in which the pressure of the water is proportional to the amount of water and gas in the bottle. Note this is different from the exhaust velocity being proportional to the mass of the rocket plus remaining fuel. Hint: it will help to present the derivation of Equation (1) in your own words so you understand where it comes from.

Plot the maximum velocity change as a function of fuel to rocket mass ratio, $(m_i - m_{dry})/m_{dry}$ where m_{dry} is the mass of the rocket with no fuel. Compare to the constant v_e rocket equation, by plotting the same function based on Equation (1) as well.

2. [1 percentage point] Determine the final momentum of the rocket after all fuel is expended and the displacement at the moment the fuel is expended. Compare the v_e=constant case to the case solved in Problem 1.